Name: ______________________ **Date:** ___________________

Math 10F&PC Chapter 4 Roots and Powers

4.4 - Fractional Exponents and Radicals

$10^5 = 100000$
$10^4 = 10000$
$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = \frac{1}{10} = 0.1$
$10^{-2} = \frac{1}{100} = 0.01$

Focus: Relate rational exponents and radicals.

Given that 4^1= __4__ and 4^0= __1__, what do you think $4^{1/2}$ will equal?
$4^{\frac{1}{2}} = 2$

Using your calculator, determine the value of each of the following.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$ same $\sqrt{1}$
4	$4^{\frac{1}{2}} = 2$ $\sqrt{4}$
9	$9^{\frac{1}{2}} = 3$ $\sqrt{9}$
16	$16^{\frac{1}{2}} = 4$ $\sqrt{16}$
25	$25^{\frac{1}{2}} = 5$ $\sqrt{25}$

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = 1$ $\sqrt[3]{1}$
8	$8^{\frac{1}{3}} = 2$ $\sqrt[3]{8}$
27	$27^{\frac{1}{3}} = 3$ $\sqrt[3]{27}$
64	$64^{\frac{1}{3}} = 4$ $\sqrt[3]{64}$
125	$125^{\frac{1}{3}} = 5$ $\sqrt[3]{125}$

This indicates that:

- Raising a number to the exponent $\frac{1}{2}$ is equivalent to taking the ***square root*** of the number.
- Raising a number to the exponent $\frac{1}{3}$ is equivalent to taking the ***cube root*** of the number

In grade 9, we learned exponent rules, including: To multiply powers with the same base, add the exponents.......... $a^m \cdot a^n = a^{m+n}$ Ex: $7^3 \times 7^5 = 7^{3+5} = 7^8$

So……… $5^2 \times 5^3 = 5^{2+3} = 5^5$

We can extend this prior knowledge to this new unit. We are going to look at powers with fractional exponents with the numerator 1. $7^{\frac{1}{3}} \times 7^{\frac{1}{3}} = 7^{\frac{1}{3}+\frac{1}{3}} = \boxed{7^{\frac{2}{3}}}$

$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^1 = 5$ and $\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$ you can clearly see that $5^{\frac{1}{2}} = \sqrt{5}$

RULE: Powers with Rational Exponents with a Numerator of 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}, \text{ where } n \neq 0$$

$4^{\frac{1}{2}} = \sqrt[2]{4} = 2$ $-6^2 = -36$ $(-6)^2 = +36$

Example 1: Positive Rational Exponents

a) $27^{\frac{1}{3}} = \sqrt[3]{27} = \sqrt[3]{3.3.3} = \boxed{3}$

b) $49^{\frac{1}{2}} = \sqrt{49} = \boxed{7}$

c) $-64^{\frac{1}{2}} = -\sqrt{64} = \boxed{-8}$

You try: a) $1000^{\frac{1}{3}}$ $= \sqrt[3]{1000}$ $= \boxed{10}$ b) $0.25^{\frac{1}{2}}$ $= \sqrt{0.25}$ $= \boxed{0.5}$ c) $(-8)^{\frac{1}{3}}$ $= \sqrt[3]{-8}$ $= \boxed{-2}$

A fraction can be written as a decimal that either terminates or repeats. When we use our calculator to determine fractional exponents we can calculate $32^{\frac{1}{5}}$ in two ways:

$32^{\frac{1}{5}}$

x^y ^ $32^\wedge(\frac{1}{5})$ inter $= \boxed{2}$ $32^\wedge 0.2 = \boxed{2}$

$32^{\frac{3}{5}} = 8$

$32^{\frac{2}{5}} = 4$

$32^{\frac{1}{5}} = 2$

Remember: ***when entering a fractional exponent into your calculator, you must use brackets!!!***

For situations when the numerator of your fractional exponent is not 1, we need to recall another exponent law: $(a^n)^m = a^{n.m}$

Power $(3^2)^3 = 3^{2\times 3} = 3^6$

So…… $6^{\frac{2}{3}} = 6^{\frac{1}{3}.2}$ $(\frac{1}{3}).2 = \frac{2}{3}$

RULE: Powers with Rational Exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\sqrt[n]{x}^{\,m}$

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Examples: $\sqrt{\ }$ $\frac{2}{3}$

1. Write $40^{\frac{2}{3}}$ in radical form $= \sqrt[3]{40^2}$ $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = 2^2 = \boxed{4}$ $= (\sqrt[3]{8})$
2. Write $\sqrt{3^5}$ in exponential form. $3^{\frac{5}{2}}$
3. Write $(\sqrt[3]{25})^2$ in exponential form. $25^{\frac{2}{3}}$

Changing Forms: *1. Radical* ⟶ *Exponential*

a. $\sqrt[3]{x^5}$ $= x^{\frac{5}{3}}$ b. $\sqrt{y^5}$ $= y^{\frac{5}{2}}$

c. $\sqrt{x}$ $= x^{\frac{1}{2}}$ d. $\sqrt[5]{y^3}$ $= y^{\frac{3}{5}}$

Example

$\sqrt[4]{20^5} = 20^{\frac{5}{4}}$

Changing Forms: *2. Exponential* ⟶ *Radical*

a. $x^{\frac{2}{7}}$ $= \sqrt[7]{x^2}$ b. $y^{\frac{1}{5}}$ $= \sqrt[5]{y}$

$0.2 = \frac{1}{5}$

c. $y^{\frac{3}{2}}$ $= \sqrt[2]{y^3}$ d. $(2x)^{0.2}$ $= \sqrt[5]{2x}$

$(2x)^{\frac{1}{5}} =$

e. $(3x^3)^{\frac{2}{7}}$ $= \sqrt[7]{(3x^3)^2}$ $= \sqrt[7]{9x^6}$ $\neq \sqrt[7]{3x^6}$ X

y^x $\wedge$

$x\sqrt{}$ 3^7

Evaluating Radicals and Exponents

$3 \; {}^{nd} \; {}^{x}\!\sqrt{73} =$

$73 \; {}^{nd} \; {}^{x}\!\sqrt{} \; 3$

A. Evaluate to the nearest tenth. Use your calculator.

1. $\sqrt[3]{73} = \boxed{4.2}$

2. $\sqrt[4]{25} = \boxed{2.2}$

3. $\sqrt[100]{15\,230} = \boxed{1.1}$

4. $\left(\sqrt[9]{55}\right)^{-2} = 55^{-\frac{2}{9}} = \boxed{0.4}$

5. $4\left(\sqrt[5]{216}\right) = 4\left(216^{\frac{1}{5}}\right) = \boxed{11.7}$

6. $-\sqrt[5]{231} = -231^{\frac{1}{5}} = -2.96 = \boxed{-3.0}$
   $= -2.9697$

7. $-2\sqrt[3]{-58} - 3\sqrt[5]{21} = -(-7.74) - (5.51)$
   $-(-7.74) - 5.51 = 7.74 - 5.51 = \boxed{2.2}$

stop here

Quiz 4.1 ~ 4.3

B. Evaluate. (Exact) Can all be done on your calculator.

$8 \; y^x \left(\frac{2}{3}\right)$ - int

1. $8^{\frac{2}{3}} = \sqrt[3]{8^2}$
   $= \boxed{4}$

2. $64^{\frac{3}{2}}$ same $\sqrt[2]{64^3}$
   $= \boxed{512}$

3. $(-27)^{\frac{1}{3}}$ same $\sqrt[3]{-27}$
   $= \boxed{-3}$

4. $-4^{\frac{-3}{2}}$ Same as $-\sqrt{4^{-3}}$
   $= \boxed{-\frac{1}{8}}$

0.004

5. $81^{-1.25}$ For Ex. NC
   $81^{-\frac{5}{4}} = \sqrt[4]{81^{-5}} = \sqrt[4]{\frac{1}{81^5}} = \sqrt[4]{\frac{1}{81^4} \times \frac{1}{81^1}} = \frac{1}{81}\sqrt[4]{\frac{1}{81}} = \frac{1}{81}\sqrt[4]{\frac{1}{3^4}} = \frac{1}{81} \times \frac{1}{3}$
   $= \boxed{\frac{1}{254}}$

6. $(0.0009)^{\frac{1}{2}}$

7. $\left(\frac{27}{8}\right)^{\frac{2}{3}} = 2.25$

8. $\left(\frac{4}{9}\right)^{-1.5} = 3.375$

9. $5^{\frac{2}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{2}{3}+\frac{1}{3}} = 5^{\frac{3}{3}}$
   $= \boxed{5}$

10. $\left(\sqrt[6]{25}\right)^3 = 5$

11. $(0.008)^{\frac{-2}{3}} = 25$

12. $32^{0.4} = 4$

$42^{\frac{5}{4}}$

a- $\sqrt[5]{42^4}$

b- $\left(\sqrt[5]{42}\right)^4$

c- $\sqrt[4]{42^5}$

d- $\left(\sqrt[4]{42}\right)^5$

Example: Write $42^{\frac{5}{4}}$ as a radical. $\sqrt[4]{42^5}$ $\left(\sqrt[4]{42}\right)^5$

a. $\sqrt[5]{42^4}$ $= 42^{\frac{4}{5}}$

b. $\left(\sqrt[4]{42}\right)^5$ $= 42^{\frac{5}{4}}$

c. $\sqrt[125]{42}$ $42^{\frac{1}{125}}$

d. $\left(\sqrt[5]{42}\right)^4$ $= 42^{\frac{4}{5}}$

Example: Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, *b* kilograms, of a mammal with body mass *m* kilograms. Estimate the brain mass of a mammal with body mass 276 kg.

$m = 276$

$b = ?$

$b = 0.01(276)^{\frac{2}{3}}$

$b = 0.4239$ kg

Example: Evaluate $\left(\frac{125}{8}\right)^{\frac{4}{3}}$. $= \dfrac{(125)^{\frac{4}{3}}}{(8)^{4/3}} = 39.06$

same as $\left(\frac{125}{8}\right)^{\frac{4}{3}} = \boxed{39.06}$

Example: Evaluate $\left(-\frac{243}{32}\right)^{0.8}$. $= \left(-\dfrac{243^{0.8}}{32^{0.8}}\right)$ same as $\left(\frac{-243}{32}\right)^{0.8}$

$= -5.0625$

Assignment ***page 227 #3–7, 10–12, 15, 18–21***